Math 115

Summer 2017

Lecture 3

Ratio of a to b is $\frac{a}{b}$, then reduce.

Ratio of 15 to 40

$$\frac{15}{40} = \frac{5 \cdot 3}{5 \cdot 8} = \frac{3}{8} \Rightarrow$$ It can be written as follow

$$15:40 = 3:8$$

Find the ratio of 2.5 to 75

$$2.5:75 = \frac{2.5}{75} = \frac{2.5(10)}{75(10)} = \frac{25}{75 \cdot 10} = \frac{25 \cdot 1}{25 \cdot 3 \cdot 10}$$

$$= \frac{1}{30}$$

$$= 1:30$$

Find the ratio of .5 to .55

$$\frac{.5}{.55} = \frac{.5(100)}{.55(100)} = \frac{50}{55} = \frac{5\cdot 10}{5\cdot 11} = \frac{10}{11} = 10:11$$

Find the ratio of $3\frac{1}{3}$ to $\frac{5}{6}$.

$$\frac{3\frac{1}{3}}{\frac{5}{6}} = 3\frac{1}{3} \div \frac{5}{6} = \frac{10}{3} \div \frac{5}{6} = \frac{\cancel{10}^{2}}{\cancel{3}_{1}} \cdot \frac{\cancel{6}^{2}}{\cancel{5}_{1}} = \frac{4}{1} = 4:1$$

Keep it

Find the ratio of {Some number} and {3 less than twice the number}

Ans. in fraction.

$$\frac{x}{2x-3}$$

When two ratios are equal, we have a Proportion.

$$\frac{a}{b} = \frac{c}{d} \Leftarrow \text{Proportion}$$

To verify a true proportion, we will perform Cross-Multiplication

$$\frac{a}{b} \quad \frac{c}{d}$$

$$ad = bc$$

Is $\frac{8.5}{12} = \frac{5}{7}$?

$12.5 \stackrel{?}{=} (8.5)7$ → $60 \stackrel{?}{=} 59.5$

False → Not a true Proportion.

True Proportion or not:

$$\frac{3\frac{1}{4}}{16} = \frac{6}{29\frac{7}{13}}$$

To verify $\Rightarrow$ Cross-Multi.

$$3\tfrac{1}{4} \cdot 29\tfrac{7}{13} \stackrel{?}{=} 16 \cdot 6$$

$$\frac{\cancel{13}}{4} \cdot \frac{384}{\cancel{13}} \stackrel{?}{=} 96$$

$$\frac{384}{4} \stackrel{?}{=} 96$$

$$96 = 96 \checkmark$$

It is a true Proportion.

How to Solve a proportion eqn:

1) Cross-Multiply

2) Solve

Ex: Solve $\frac{x-1}{x+4} = \frac{1}{2}$

Cross-Multiply $2(x-1) = 1(x+4)$

$2x - 2 = x + 4$

$2x - x = 4 + 2$

$\boxed{x = 6}$

$\{6\}$

Solve: $\frac{3x+5}{x-3} = \frac{5}{2}$

Cross-Multiply

$2(3x+5) = 5(x-3)$

Distribute

$6x + 10 = 5x - 15$

$6x - 5x = -15 - 10$

$x = -25$

$\{-25\}$

Solve

$\frac{2x+7}{3x-4} \quad \frac{2}{3}$

$3(2x+7) = 2(3x-4)$

$6x + 21 = 6x - 8$

$6x - 6x = -8 - 21 \Rightarrow 0 = -29$ false

$\emptyset$ No Soln.

Mary used 2.5 cups of sugar to bake 15 muffins. How many cups of sugar for 80 muffins?

$\frac{2.5 \text{ cups}}{15 \text{ muf.}} = \frac{x \text{ cups}}{80 \text{ muf.}}$

About 13.3 cups of sugar.

$\frac{2.5}{15} = \frac{x}{80}$

$15x = 2.5(80)$

$x = \frac{2.5(80)}{15}$

$x = 13.\overline{3}$

A 5.5 ft tall person has a shadow of 12 ft. long.

At the same time, A tall tree has a shadow of 40 ft. How tall is the tree?

$$\frac{5.5 \text{ ft tall}}{12 \text{ ft. shadow}} = \frac{x \text{ ft. tall}}{40 \text{ ft. shadow}}$$

$$\frac{5.5}{12} = \frac{x}{40} \Rightarrow 12x = 40(5.5)$$

$$x = \frac{40(5.5)}{12}$$

$$x = 18.\overline{3}$$

About 18 ft tall

Mike is working for the fishing dept. in LA county. His project is to estimate the number of fish at East LA Lake.

He caught 20 fish, tagged them all, and released them back in the lake.

Next day, he caught 25 fish, but only 4 had tags. Use ratio & Prop. to estimate the # of fish in East LA Lake.

$$\frac{x \text{ fish}}{20 \text{ tags}} = \frac{25 \text{ fish}}{4 \text{ tags}}$$

$$4x = 20(25)$$

$$x = \frac{20(25)}{4}$$

$$x = 125$$

125 fish

Lisa is traveling between two cities. She notices that the distance between them on the map is 12.5 inches. The map uses scales of 1.25 inches for every 50 miles. Find the actual distance between two cities.

$$\frac{x \text{ miles}}{12.5 \text{ inches apart}} = \frac{50 \text{ miles}}{1.25 \text{ inches Scale}} \qquad \frac{x}{12.5} = \frac{50}{1.25}$$

$$1.25x = 12.5(50)$$

$$x = \frac{12.5(50)}{1.25} \qquad x = 500$$

They are 500 miles apart.

Basic Percent

$$8\% = \frac{8}{100} = \frac{4.2}{4.25} = \boxed{\frac{2}{25}}$$

$\% \Rightarrow \frac{1}{100}$

$\Rightarrow .01$

$$8\% = 8(.01) = \boxed{.08}$$

$$.5\% = \frac{.5}{100} = \frac{.5(10)}{100(10)} = \frac{\cancel{5}^{1}}{100(\cancel{10}^{2})} = \boxed{\frac{1}{200}}$$

$$.5\% = .5(.01) = \boxed{.005} = \boxed{0.005}$$

Convert 2.5% to a reduced fraction.

$$2.5\% = \frac{2.5}{100} = \frac{25}{100(10)} = \boxed{\frac{1}{40}}$$

$$\frac{\cancel{25}\cdot 1}{\cancel{25}\cdot 4\cdot 10}$$

Convert 125% to a reduced fraction and in decimal notation.

$125\% = \frac{125}{100} = \frac{25 \cdot 5}{25 \cdot 4} = \boxed{\frac{5}{4}}$

$125 = 5 \cdot 5 \cdot 5$

$100 = 2 \cdot 2 \cdot 5 \cdot 5$

$125\% = 125(.01) = \boxed{1.25}$

$1.25\% = 1.25(.01)$

$= \boxed{.0125}$

Convert 1.25% to reduced fraction and to decimal.

$1.25\% = \frac{1.25}{100} = \frac{1.25(100)}{100(100)} = \frac{125}{100(100)} = \boxed{\frac{1}{80}}$

(1, 5, 25, 20, 20, 4)

Using Proportions to Solve Basic Percent

$$\frac{P}{100} = \frac{\text{Part}}{\text{whole}}$$

"whole comes after of"

12% of what number is 60?

$$\frac{P}{100} = \frac{\text{Part}}{\text{whole}}$$

$$\frac{12}{100} = \frac{60}{x}$$

$12x = 100(60)$

$x = \frac{100(60)}{12}$ (5, 12, 1)

$\boxed{x = 500}$

12% of 500 is 60.

2.5% of what number is 175?

$\frac{P}{100} = \frac{\text{Part}}{\text{whole}}$

$\frac{2.5}{100} = \frac{175}{x}$

$2.5x = 100(175)$

$x = \frac{100(175)}{2.5}$

$x = 7000$

2.5% of 7000 is 175.

8% of 4500 is what number?

$\frac{P}{100} = \frac{\text{Part}}{\text{whole}}$

$\frac{8}{100} = \frac{x}{4500}$

$100x = 8(4500)$

$x = \frac{8(4500)}{100}$

$x = 360$

8% of 4500 is 360.

What percent of 1200 is 96?

$$\frac{P}{100} = \frac{\text{Part}}{\text{whole}}$$

$$\frac{P}{100} = \frac{96}{1200}$$

$$1200P = 100(96)$$

$$P = \frac{100(96)}{1200} \qquad P = 8$$

8% of 1200 is 96.

What percent is 19 out of 40?

$$\frac{P}{100} = \frac{\text{Part}}{\text{whole}}$$

$$\frac{P}{100} = \frac{19}{40}$$

$$40P = 100(19)$$

$$P = \frac{100(19)}{40}$$

$$P = 47.5$$

47.5% of 40 is 19.

or

19 out of 40 is 47.5%

32 of 72 is what percent?

$$\frac{P}{100} = \frac{\text{Part}}{\text{whole}}$$

$$\frac{P}{100} = \frac{32}{72}$$

$$72P = 100(32)$$

$$P = \frac{100(32)}{72}$$

$P = 44.\overline{4}$

32 of 72 is $44.\overline{4}\%$.

75% of what number is 465?

$$\frac{75}{100} = \frac{465}{x} \qquad 75x = 100(465) \qquad x = 620$$

75% of 620 is 465

what number is 15.2% of 90?

$$\frac{15.2}{100} = \frac{x}{90} \qquad 100x = 90(15.2) \qquad x = 13.68$$

13.68 is 15.2% of 90.

What percent of 98 is 106.33?

$\frac{P}{100} = \frac{106.33}{98}$ $\quad 98P = 100(106.33)$

$P = 108.5$

108.5% of 98 is 106.33

12.5% of what is 937.5?

$\frac{12.5}{100} = \frac{937.5}{x}$ $\quad 12.5x = 100(937.5)$

$x = 7500$

12.5% of 7500 is 937.5.

WP 2 & WP 3 Due

Thursday @ 6:00 AM.

1) one side only

2) No need to write the actual problem.

3) $\begin{array}{c|c} 1 & 2 \\ \hline 3 & 4 \end{array}$

4) Place your final answer where it says Ans.___

5) Staple papers in order.

SG 2 due Thursday

Solve:

$$4(3x-2)-5=2x+12$$

$$12x-8-5=2x+12$$

$$12x-13=2x+12$$

$$12x-2x=12+13$$

$$10x=25$$

$$\boxed{x=2.5}$$

$$\{2.5\}$$

Solve

$$.25x+.1(2x+1)=3.25$$

$$.25x+.2x+.1=3.25$$

$$.45x=3.25-.1$$

$$.45x=3.15$$

$$x=\frac{3.15}{.45}$$

$$\boxed{x=7} \Rightarrow \{7\}$$

Solve $\frac{3}{4}x - \frac{2}{5} = \frac{4}{5}x + \frac{1}{2}$

LCD = 20

$20 \cdot \frac{3}{4}x - 20 \cdot \frac{2}{5} = 20 \cdot \frac{4}{5}x + 20 \cdot \frac{1}{2}$

$15x - 8 = 16x + 10$

$15x - 16x = 10 + 8$

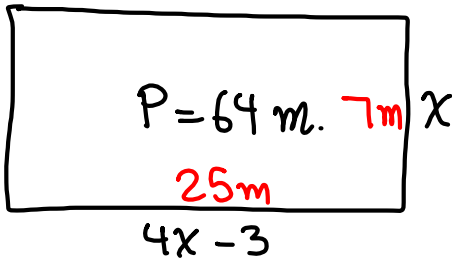
$-1x = 18$

$x = \frac{18}{-1}$

$x = -18$

$\{-18\}$

Find the dimensions of the following Shape:



$4(7) - 3 = 25$

Dimensions are 7m by 25m.

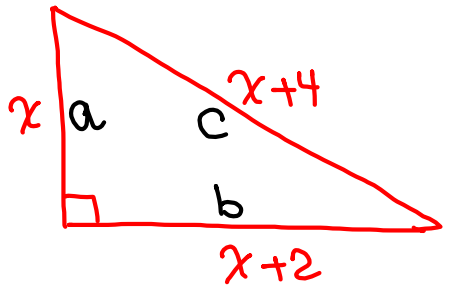
$P = 2L + 2W$

$P = 64$

$2L + 2W = 64$

$2(4x-3) + 2(x) = 64$

$x = 7$



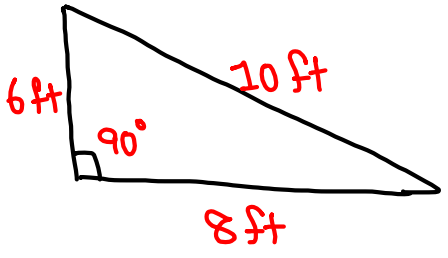
Find all three sides, and the area if the perimeter is 24 ft.

$P = a + b + c$

$x + x+2 + x+4 = 24$

$3x + 6 = 24$

$x = 6$



$A = \frac{bh}{2} = \frac{8 \cdot 6}{2} = 24$

Three sides are 6 ft, 8 ft, and 10 ft.
The area is $24\ ft^2$.

Type of linear Equations:

1) Conditional ⇔ Exactly one Soln.

2) Identity ⇔ Infinitely Many Solns.

3) Contradiction ⇔ No Soln.

Solve & identify the type of eqn:

1) $-3(2x+7)+4(x-2)=13$

$-6x-21+4x-8=13$

$-2x-29=13$

$-2x=42$

$\boxed{x=-21} \Rightarrow \{-21\}$

Exactly one Soln $\Rightarrow$ Conditional eqn.

2) $5(x-3)-2(x+8)=3x+20$

$5x-15-2x-16=3x+20$

$3x-31=3x+20$

$3x-3x=20+31$

$0=51$ False $\Rightarrow \emptyset$

No Soln $\Rightarrow$ Contradiction.

3) $3(4x+7)-6(2x-10)=81$

$12x+21-12x+60=81$

$81=81$ True

infinitely Many Solns

Identity

There are 52 people in a meeting. The number of males is 1 fewer than twice the number of females. How many of each?

More

$2x+1+x=52$

$3x=51$

$x=17$

17 Females & 35 Males

Males + Females = 52

$2x-1+x=52$

$3x-1=52$

$3x=53$

$x=17.\overline{6}$

Not practical
No Soln.

Eqns with more than one variable are called formulas.

$P = a + b + c$ Perimeter of a triangle

Solve for a

"Isolate a"

$\boxed{P - b - c = a}$

$P = 2L + 2W$ Perimeter of a rectangle

Solve for L

"Isolate L"

$P - 2W = 2L$

$\boxed{\frac{P - 2W}{2} = L}$

$A = \frac{bh}{2}$ Solve for h.

LCD = 2

$2A = \cancel{2} \cdot \frac{bh}{\cancel{2}}$

$\Rightarrow 2A = bh$

$\boxed{\frac{2A}{b} = h}$

Solve for y:

$3x + y = 6$

$y = -3x + 6$ or $y = 6 - 3x$

Solve for y:

$-5x + 2y = -8$

$2y = 5x - 8$

$y = \frac{5}{2}x - \frac{8}{2}$

$y = \frac{5}{2}x - 4$

or

$y = \frac{5x - 8}{2}$

$\frac{2}{5}x - \frac{1}{2}y = 1$

Hint: use LCD to clear fractions.

Solve for y

LCD = 10

$\frac{2}{10} \cdot \frac{2}{5}x - \frac{5}{10} \cdot \frac{1}{2}y = 10 \cdot 1$

$4x - 5y = 10$

$-5y = -4x + 10$

$y = \frac{-4}{-5}x + \frac{10}{-5}$

$y = \frac{4}{5}x - 2$

or

$y = \frac{-4x + 10}{-5}$

1) Evaluate $\dfrac{x^2-5x}{x^2-25}$ for $x=5$.

$= \dfrac{5^2-5(5)}{5^2-25} = \dfrac{25-25}{25-25} = \dfrac{0}{0}$ Indeterminate

2) Simplify $-3(2x^2+5x-2)+6x^2+16x-6$

$= -6x^2 - 15x + 6 + 6x^2 + 16x - 6$

$= 1x = \boxed{x}$

$(2x)^2 \neq 2x^2$

Simplify:

$\left(\dfrac{3}{5}-\dfrac{1}{2}\right)^2 \div \sqrt{\dfrac{1}{100}} - \dfrac{3}{10}$

LCD = 10

$= \left(\dfrac{3\cdot 2}{5\cdot 2}-\dfrac{1\cdot 5}{2\cdot 5}\right)^2 \div \sqrt{\dfrac{1}{100}} - \dfrac{3}{10}$

$= \left(\dfrac{6}{10}-\dfrac{5}{10}\right)^2 \div \sqrt{\dfrac{1}{100}} - \dfrac{3}{10}$

$= \left(\dfrac{1}{10}\right)^2 \div \sqrt{\dfrac{1}{100}} - \dfrac{3}{10} =$

$= \dfrac{1}{100} \div \dfrac{1}{10} - \dfrac{3}{10}$

$= \dfrac{1}{\cancel{100}_{10}} \cdot \dfrac{\cancel{10}^{1}}{1} - \dfrac{3}{10}$

$= \dfrac{1}{10} - \dfrac{3}{10}$

$= \dfrac{-2}{10} = \boxed{\dfrac{-1}{5}}$

Due Tomorrow @ 6:00: wp2, wp3

" " @ 10:25: SG2

Exam 1: Tuesday

We will finish ch. 2 Tomorrow.